Tail Value-at-Risk

[Nematrian website page: TailValueAtRisk, © Nematrian 2015]

The Tail Value-at-Risk, TVaR, of a portfolio $TVaR_\alpha(X)$ is defined as the expected outcome (loss), conditional on the loss exceeding the Value-at-Risk (VaR), of the distribution.
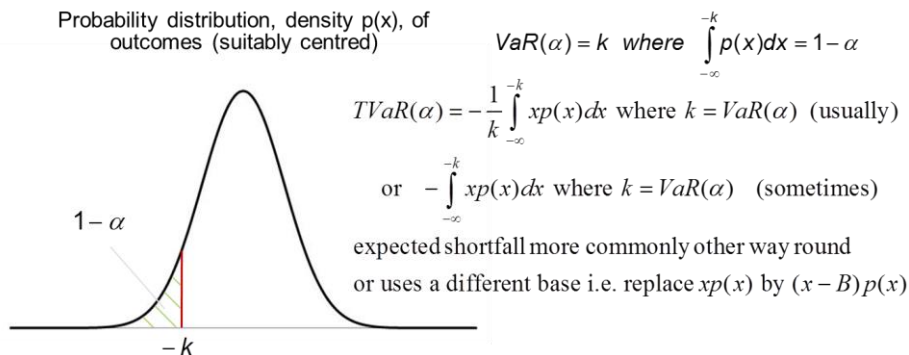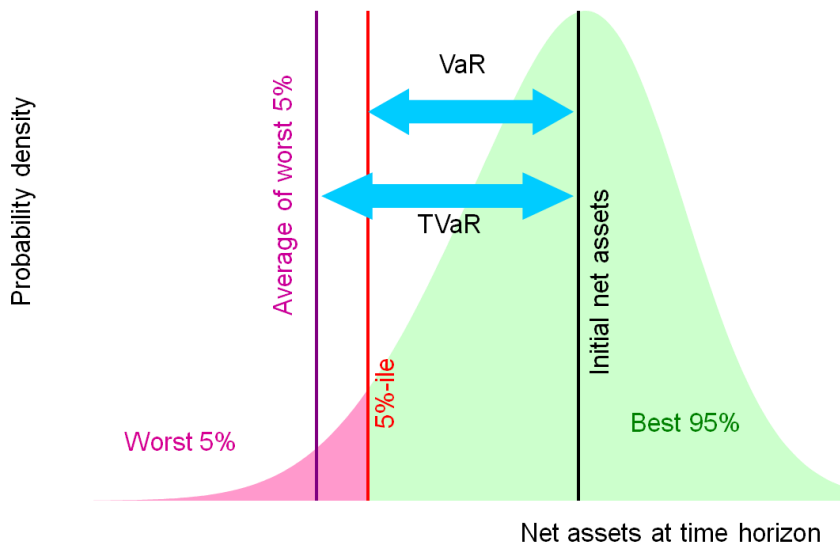
Where the support of the distribution is continuous the VaR with confidence level α is usually defined as follows:

$$Prob(x \leq -VaR_\alpha) = 1 - \alpha$$

The corresponding Tail Value-at-Risk would then be defined as:

$$TVaR_\alpha = -\frac{1}{1-\alpha} \int_{-\infty}^{-VaR_\alpha} xf(x)dx$$

Visually the difference between VaR and Tail VaR may be seen in either of the following charts:

VaR is not (in general) a coherent risk measure, whilst TVaR is. VaR is arguably more shareholder focused and TVaR more regulator/customer focused, see VaR versus TVaR mindsets.

Writers use Tail VaR (TVaR) and Conditional VaR (CVaR) largely interchangeably, usually with the same loss trigger as the quantile level that would otherwise be applicable if the focus was on VaR. Occasionally, TVaR and/or CVaR are differentiated, with one being expressed in terms of the loss *beyond* the VaR rather than below zero. However, such a definition inherits some technical weaknesses attributable to VaR (i.e. that it no longer exhibits diversification properties we might 'expect' a risk measure to exhibit.

Another term that means much the same thing is Conditional Tail Expectation (CTE), although perhaps this is more likely to refer to the right tail of a distribution rather than the left tail, i.e. it might focus on upside rather than downside, and the bound beyond which it is calculated may not be expressed in a VaR-like form.

Expected Shortfall has a similar meaning, but might use a trigger level set more generically, e.g. it might include all returns below some level (e.g. zero) and more commonly might no longer contain the $1/(1-\alpha)$ multiplier included in the definition of TVaR a above.

If several different risk exposures are contributing to the overall TVaR then it often becomes important to identify the contribution each is making to the total. This can be done using marginal Tail Value-at-Risk (or marginal TVaR).