Marginal Value-at-Risk (Marginal VaR)

Suppose we have a set of n risk factors which we can characterise by an n-dimensional vector $\mathbf{x} = (x_1, \dots, x_n)^T$. Suppose that the (active) exposures we have to these factors are characterised by another n-dimensional vector, $\mathbf{a} = (a_1, \dots, a_n)^T$. Then the aggregate exposure is $\mathbf{a}.\mathbf{x}$.

The *Value-at-Risk* of the portfolio of exposures $\mathbf{a}$ at confidence level α, $VaR_\alpha(\mathbf{a})$, is usually defined to be the value such that $Pr\left(\mathbf{a}.\mathbf{x} \leq -VaR_\alpha(\mathbf{a})\right) = 1 - \alpha$.

The Marginal Value-at-Risk, $MTVaR_{\alpha,i}(\mathbf{a})$, is the sensitivity of $TVaR_\alpha(\mathbf{a})$ to a small change in i'th exposure. It is therefore:

$$MVaR_{\alpha,i}(\mathbf{a}) = \frac{\partial VaR_\alpha(\mathbf{a})}{\partial a_i}$$

Because Value-at-Risk is (first-order) homogeneous (for a continuous probability distribution) it satisfies the Euler capital allocation principle and hence:

$$VaR_\alpha(\mathbf{a}) = \sum_{i=1}^{n} a_i MVaR_{\alpha,i}(\mathbf{a})$$

If the risk factors are multivariate normally distributed then $MVaR_{\alpha,i}(\mathbf{a})$ can be expressed using a relatively simple formula, see here.

The Marginal Value-at-Risk is sometimes called the incremental value at risk (perhaps because a leading software vendor uses the latter terminology). More usually incremental value at risk is defined as follows, where $\mathbf{a}^*$ is the same as $\boldsymbol{a}$ except that it has 0 for its i'th entry.

$$IVaR_{\alpha,i}(\mathbf{a}) = VaR_\alpha(\mathbf{a}) - VaR_\alpha(\mathbf{a}^*)$$