# Marginal Tail Value-at-Risk (Marginal TVaR)



Suppose we have a set of $n$ risk factors which we can characterise by an $n$-dimensional vector $\mathbf{x} = (x_1, \dots, x_n)^T$. Suppose that the (active) exposures we have to these factors are characterised by another $n$-dimensional vector, $\mathbf{a} = (a_1, \dots, a_n)^T$. Then the aggregate exposure is $\mathbf{a}.\mathbf{x}$.

The *Value-at-Risk* of the portfolio of exposures $\mathbf{a}$ at confidence level $\alpha$, $VaR_\alpha(\mathbf{a})$, is usually defined to be the value such that $Pr\left(\mathbf{a}.\mathbf{x} \le -VaR_\alpha(\mathbf{a})\right) = 1 - \alpha$.

The Tail Value-at-Risk is defined as:

$$TVaR_\alpha = -\frac{1}{1-\alpha} \int_{-\infty}^{-VaR_\alpha} xf(x)dx$$

The Marginal Tail Value-at-Risk, $MTVaR_{\alpha,i}(\mathbf{a})$, is the sensitivity of $TVaR_\alpha(\mathbf{a})$ to a small change in $i$'th exposure. It is therefore:

$$MTVaR_{\alpha,i}(\mathbf{a}) = \frac{\partial TVaR_\alpha(\mathbf{a})}{\partial a_i}$$

Because Tail Value-at-Risk is (first-order) homogeneous (for a continuous probability distribution) it satisfies the Euler capital allocation principle and hence:

$$VaR_\alpha(\mathbf{a}) = \sum_{i=1}^{n} a_i MVaR_{\alpha,i}(\mathbf{a})$$

If the risk factors are multivariate normally distributed then $MVaR_{\alpha,i}(\mathbf{a})$ can be expressed using a relatively simple formula, see here.