Elliptical distributions

An elliptical distribution is a multivariate distribution whose (multivariate) characteristic function, $\phi(\mathbf{t})$, $\mathbf{t}$ being a vector with the same number of entries as there are dimensions for the distribution, of the form:

$$e^{i\mathbf{t}^T\boldsymbol{\mu}}\Psi(\mathbf{t}^T\mathbf{V}\mathbf{t})$$

where $\boldsymbol{\mu}$ is a specified variable, and $\mathbf{V}$ is a positive definite matrix.

If the distribution has a probability density function then it will take the form:

$$f(\mathbf{x}) = k.\, g\big((\mathbf{x}-\boldsymbol{\mu})^T\mathbf{V}^{-1}(\mathbf{x}-\boldsymbol{\mu})\big)$$

where k is a scale factor, $\mathbf{x}$ is an n-dimensional random vector with median vector $\boldsymbol{\mu}$ (which is also the mean vector, if the latter exists), is a positive definite matrix which is proportional to the covariance matrix if the latter exists, and g is a function mapping non-negative real numbers to non-negative real numbers with finite area under the curve.

Perhaps the best known elliptical distributions are multivariate normal (i.e. Gaussian) distributions.