Formulae for prices and Greeks for European (vanilla) puts in a Black-Scholes world

[Nematrian website page: BlackScholesGreeksVanillaPuts, © Nematrian 2015]

See Black Scholes Greeks for notation.

Payoff, see MnBSPutPayoff

$$Payoff = \max(K - S, 0)$$

Price (value), see MnBSPutPrice

$$Price = V = -Se^{-q(T-t)}N(-d_1) + Ke^{-r(T-t)}N(-d_2)$$

Delta (sensitivity to underlying), see MnBSPutDelta

$$Delta = \frac{\partial V}{\partial S} = -e^{-q(T-t)}N(-d_1)$$

Gamma (sensitivity of delta to underlying), see MnBSPutGamma

$$Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{e^{-q(T-t)}N'(-d_1)}{S\sigma\sqrt{T-t}}$$

Speed (sensitivity of gamma to underlying), see MnBSPutSpeed

$$Speed = \frac{\partial^3 V}{\partial S^3} = -\frac{e^{-q(T-t)}N'(-d_1)\left(d_1 + \sigma\sqrt{T-t}\right)}{\sigma^2 S^2 (T-t)}$$

Theta (sensitivity to time), see MnBSPutTheta

$$Theta = \frac{\partial V}{\partial t} = -\frac{\sigma S e^{-q(T-t)}N'(-d_1)}{2\sqrt{T-t}} - qSe^{-q(T-t)}N(-d_1) + rKe^{-r(T-t)}N(-d_2)$$

Charm (sensitivity of delta to time), see MnBSPutCharm

$$Charm = \frac{\partial^2 V}{\partial S \partial t} = -qe^{-q(T-t)}N(-d_1) + e^{-q(T-t)}N'(-d_1)\left(\frac{d_2}{2(T-t)} - \frac{r-q}{\sigma\sqrt{T-t}}\right)$$

Colour (sensitivity of gamma to time), see MnBSPutColour

$$Colour = \frac{\partial^3 V}{\partial S^2 \partial t} = \frac{e^{-q(T-t)}N'(-d_1)}{S\sigma\sqrt{T-t}}\left(q + \frac{1 - d_1 d_2}{2(T-t)} + \frac{d_1(r-q)}{\sigma\sqrt{T-t}}\right)$$

Rho(interest) (sensitivity to interest rate), see MnBSPutRhoInterest

$$Rho(Interest) = \frac{\partial V}{\partial r} = -K(T-t)e^{-r(T-t)}\,N(-d_2)$$

Rho(dividend) (sensitivity to dividend yield), see MnBSPutRhoDividend

$$Rho(Dividend) = \frac{\partial V}{\partial q} = -S(T-t)e^{-q(T-t)}N(-d_1)$$

Vega (sensitivity to volatility), see MnBSPutVega*

$$Vega = \frac{\partial V}{\partial \sigma} = Se^{-q(T-t)}N'(-d_1)\sqrt{T-t}$$

Vanna (sensitivity of delta to volatility), see MnBSPutVanna*

$$Vanna = \frac{\partial^2 V}{\partial S \partial \sigma} = -\frac{d_2 e^{-q(T-t)}N'(-d_1)}{\sigma}$$

Volga (or Vomma) (sensitivity of vega to volatility), see MnBSPutVolga*

$$Volga = \frac{\partial^2 V}{\partial \sigma^2} = \frac{d_1 d_2 Se^{-q(T-t)}N'(-d_1)\sqrt{T-t}}{\sigma}$$

* Greeks like vega, vanna and Volga/vomma that involve partial differentials with respect to σ are in some sense 'invalid' in the context of Black-Scholes, since in its derivation we assume that σ is constant. We might interpret them as applying to a model in which σ was slightly variable but otherwise was close to constant for all S, t etc.. Vega, for example, would then measure the sensitivity to changes in the mean level of σ. For some types of derivatives, e.g. binary puts and calls, it can be difficult to interpret how these particular sensitivities should be understood.