Formulae for prices and Greeks for European binary calls in a Black-Scholes world

[Nematrian website page: BlackScholesGreeksBinaryCalls, © Nematrian 2015]

See Black Scholes Greeks for notation.

Payoff, see MnBSBinaryCallPayoff

$$Payoff = \begin{cases} 1 & \text{if } S > K \\ 0 & \text{if } S \leq K \end{cases}$$

Price (value), see MnBSBinaryCallPrice

$$Price = V = e^{-r(T-t)}N(d_2)$$

Delta (sensitivity to underlying), see MnBSBinaryCallDelta

$$Delta = \frac{\partial V}{\partial S} = \frac{e^{-r(T-t)}N'(d_2)}{S\sigma\sqrt{T-t}}$$

Gamma (sensitivity of delta to underlying), see MnBSBinaryCallGamma

$$Gamma = \frac{\partial^2 V}{\partial S^2} = -\frac{e^{-r(T-t)}d_1 N'(d_2)}{S^2\sigma^2(T-t)}$$

Speed (sensitivity of gamma to underlying), see MnBSBinaryCallSpeed

$$Speed = \frac{\partial^3 V}{\partial S^3} = -\frac{e^{-r(T-t)}N'(d_2)}{\sigma^2 S^3(T-t)}\left(-2d_1 + \frac{1-d_1 d_2}{\sigma\sqrt{T-t}}\right)$$

Theta (sensitivity to time), see MnBSBinaryCallTheta

$$Theta = \frac{\partial V}{\partial t} = re^{-r(T-t)}N(d_2) + e^{-r(T-t)}N'(d_2)\left(\frac{d_1}{2(T-t)} - \frac{r-q}{\sigma\sqrt{T-t}}\right)$$

Charm (sensitivity of delta to time), see {Hpl|~/MnBSBinaryCallCharm.aspx|MnBSBinaryCallCharm}

$$Charm = \frac{\partial^2 V}{\partial S \partial t} = \frac{e^{-r(T-t)}N'(d_2)}{S\sigma\sqrt{T-t}}\left(r + \frac{1-d_1 d_2}{2(T-t)} + \frac{d_2(r-q)}{\sigma\sqrt{T-t}}\right)$$

Colour (sensitivity of gamma to time), see MnBSBinaryCallColour

$$Colour = \frac{\partial^3 V}{\partial S^2 \partial t} = -\frac{e^{-r(T-t)}N'(d_2)}{S^2\sigma^2(T-t)}\left(rd_1 + \frac{2d_1 + d_2}{2(T-t)} - \frac{(r-q)}{\sigma\sqrt{T-t}} - d_1 d_2\left(\frac{d_1}{2(T-t)} - \frac{r-q}{\sigma\sqrt{T-t}}\right)\right)$$

Rho(interest) (sensitivity to interest rate), see MnBSBinaryCallRhoInterest

$$Rho(Interest) = \frac{\partial V}{\partial r} = -(T-t)e^{-r(T-t)}N(d_2) + \frac{\sqrt{T-t}}{\sigma}e^{-r(T-t)}N'(d_2)$$

Rho(dividend) (sensitivity to dividend yield), see MnBSBinaryCallRhoDividend

$$Rho(Dividend) = \frac{\partial V}{\partial q} = -(T-t)e^{-r(T-t)}\, N(d_2)$$

Vega (sensitivity to volatility), see MnBSBinaryCallVega*

$$Vega = \frac{\partial V}{\partial \sigma} = e^{-r(T-t)} N'(d_2)\frac{d_1}{\sigma}$$

Vanna (sensitivity of delta to volatility), see MnBSBinaryCallVanna*

$$Vanna = \frac{\partial^2 V}{\partial S \partial \sigma} = -\frac{e^{-r(T-t)} N'(d_2)(1 - d_1 d_2)}{S\sigma^2\sqrt{T-t}}$$

Volga (or Vomma) (sensitivity of vega to volatility), see MnBSBinaryCallVolga*

$$Volga = \frac{\partial^2 V}{\partial \sigma^2} = \frac{e^{-r(T-t)} N'(d_2)(d_1^2 d_2 - d_1 - d_2)}{\sigma^2}$$

* Greeks like vega, vanna and Volga/vomma that involve partial differentials with respect to σ are in some sense 'invalid' in the context of Black-Scholes, since in its derivation we assume that σ is constant. We might interpret them as applying to a model in which σ was slightly variable but otherwise was close to constant for all S, t etc.. Vega, for example, would then measure the sensitivity to changes in the mean level of σ. For some types of derivatives, e.g. binary puts and calls, it can be difficult to interpret how these particular sensitivities should be understood.