VaR vs Tail VaR Mindsets

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VaR vs Tail VaR Mindsets

- What is the difference between VaR ('Value-at-Risk') and TVaR ('Tail Value-at-Risk')?

- What are the underlying mindsets and which one is more suitable for capital adequacy?

- Example implications

VaR versus TVaR

Most insurers use 0.5% VaR with 1-year horizon

Probability density

Average of worst 5%

VaR

TVaR

5%-ile

Initial net assets

Worst 5%

Best 95%

Net assets at time horizon

The Actuarial Profession
making financial sense of the future
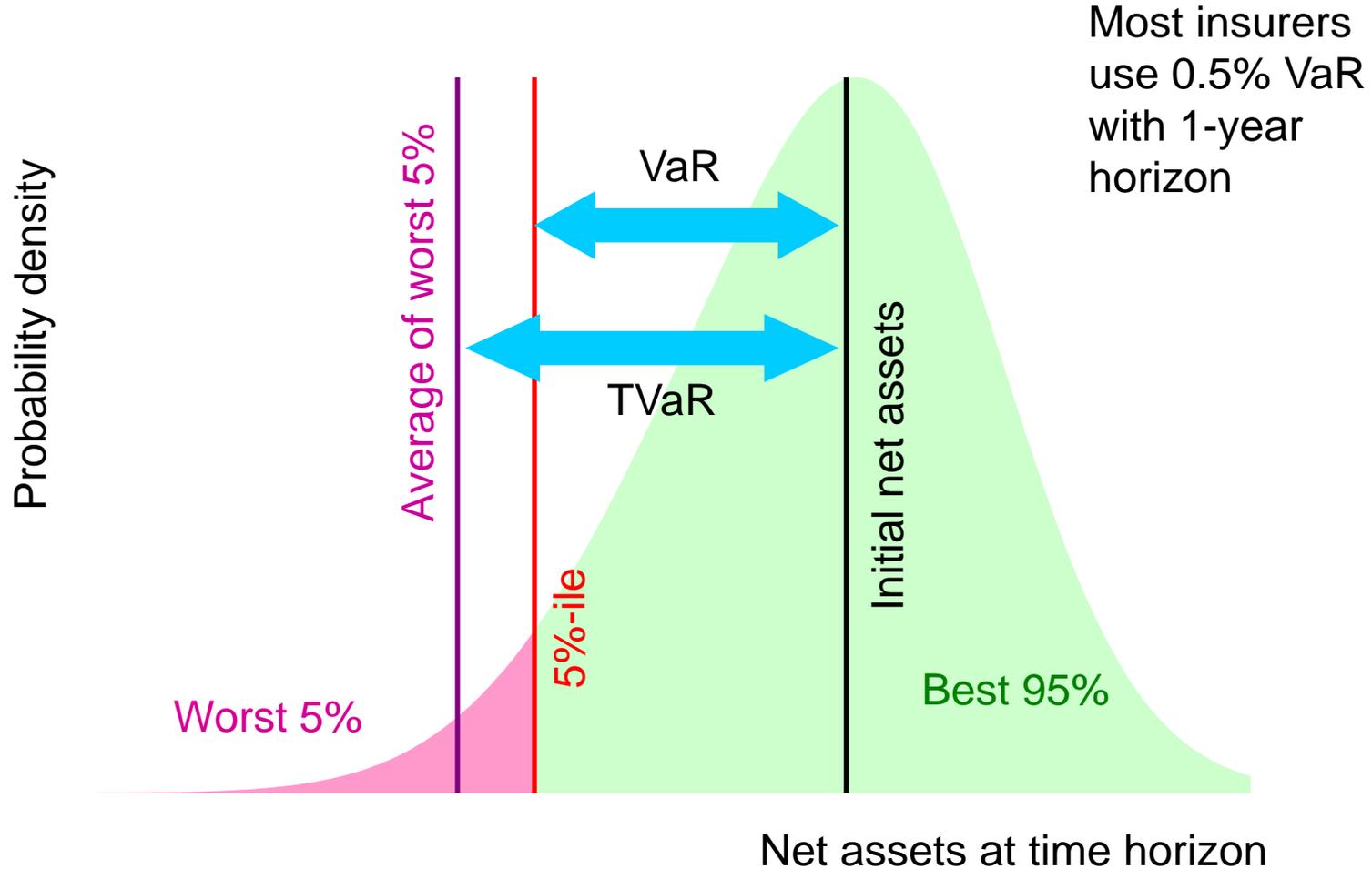
Mathematical definitions

Probability distribution, density p(x),
of outcomes (suitably centred)

$1 - \alpha$

$-k$
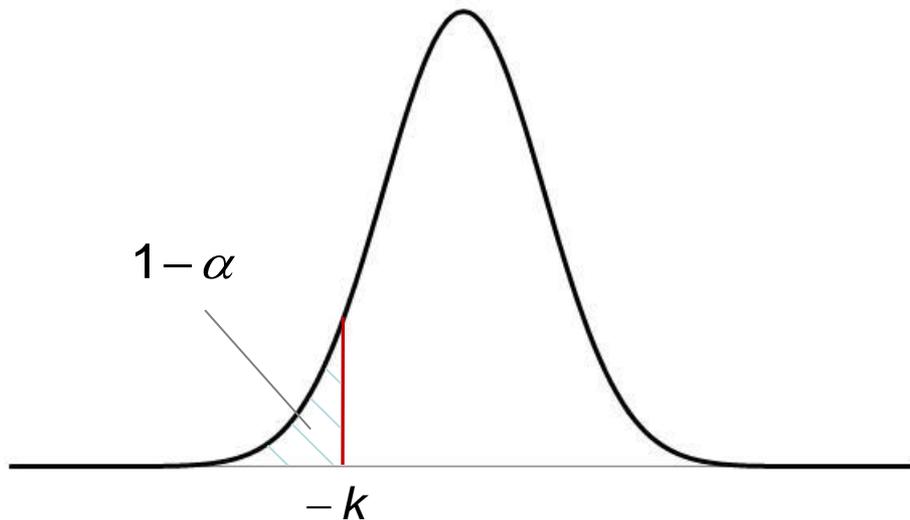
$$VaR(\alpha) = k \;\; where \;\; \int_{-\infty}^{-k} p(x)dx = 1 - \alpha$$

$$TVaR(\alpha) = -\int_{-\infty}^{-k} xp(x)dx \;\; where \;\; k = VaR(\alpha)$$

- Note difference between *p(x)* and *xp(x)* in the integrals

VaR versus TVaR (1)

- Arguments in favour of TVaR are usually expressed in relatively mathematical language

- Around the concept of *coherence*

- E.g. 99% confidence level, firm A has one exposure to a 1 in 500 risk of loss of 100m, firm B has ten (independent) exposures to 1 in 500 risks of loss of £10m

- VaR for A (=0) less than VaR for B, even though B better diversified. TVaR behaves more 'sensibly'

What are the underlying mindsets?

- Suppose we have two 'pay-offs' (business opportunities, financial outcomes, ...), C and D
  - With C, receive M if event X occurs (X has probability p, $p > 0$)
  - With D, receive $2M$ if event X occurs
- Which do we prefer?
  - D (if $M > 0$), C (if $M < 0$)
- To value a risky bond or claim we include a term like:

  Probability of default ('PD') x Loss Given Default ('LGD')

VaR vs. TVaR (2)

- VaR: focuses on the PD element alone

- TVaR: *also* takes into account the LGD

- Markets (and some parts of existing regulatory frameworks) recognise the need to take into account LGD as well as PD when valuing and assessing the riskiness of a credit sensitive instrument
  - Why don't we therefore apply it to the whole portfolio?

Shareholder vs. Policyholder vs. Regulator Perspectives (1)

- Shareholders (in a limited liability company) benefit from the 'solvency put option'
  - They largely *don't* care about size of loss *in the event of default* (i.e. the LGD)
  - Because they have already lost all that they are going to suffer
- Policyholders *do* care about the LGD
- At least they do up to the detachment point at which any further LGD gets passed on to other stakeholders
  - e.g. Government or industry-wide protection schemes (who thus in turn have an interest in the LGD)

Shareholder vs. Policyholder vs. Regulator Perspectives (2)

Risk Measure	Shareholder	Policyholder	Regulator (and equivalent stakeholders)
VaR	👍 (ignores LGD)		
Tail VaR		👍 (includes LGD)	👍 (includes LGD)

- Capital adequacy is policyholder/regulator focused
- So the VaR mindset is wrong for it
- Use of TVaR would redress the lack of focus on LGD within VaR

Example implications

- Treatment of illiquidity

- Stress testing methodologies

- Market consistent capital adequacy

Treatment of illiquidity (1)

- Two otherwise identical firms, A and B:
  - Larger line (constituting bulk of the firms' overall risk). Both A and B have the same assets and liabilities. Assumed not exposed to liquidity risk (e.g. liquid unit-linked).
  - Smaller line: involves highly illiquid liabilities (e.g. annuity book): Same liabilities. **A** invests in **illiquid** assets arguing that these best match the illiquid nature of the liabilities. **B** invests in **liquid** assets with similar cash flow timings.
- Which *should* the policyholder prefer?
  - In other words, what credit *should* we allow for the illiquidity premium potentially available on illiquid assets?

Treatment of illiquidity (2)

- Policyholder *should* (generally) prefer B to A
  - PD largely driven by non-liquidity risks, so roughly the same for both firms
  - LGD driven by what happens in the event of default
- Default will most probably be associated with forced liquidation of assets (and forced transfer of liabilities)
  - Which asset type is likely to realise more in a fire sale – a liquid one or an illiquid one?
- Possibly mitigating effects over longer time horizons

Treatment of illiquidity (3)

- Logic of matching illiquid liabilities with illiquid assets predicated on assumption that the firm is a hold-to-maturity investor

- But LGD relates to situations where the firm has typically lost its ability to hold-to-maturity

- VaR based approaches will thus miss this subtlety

- TVaR based approaches (if properly implemented) shouldn't

Stress testing methodologies

- Increasing regulatory focus on stress testing
  - Including liquidity stresses
- E.g. Reverse stress-testing or "test to destruction"
- But these again focus on the PD element
- What we ideally need is a "test beyond destruction"
  - Otherwise we will miss the LGD element
- As the FSA point out, capital is held to cover both the "going concern" and the "gone" concern situation, hence different Tiers

Market consistent capital adequacy

- When valuing a risky bond or claim there is actually a third component, i.e. the time value:

  PD x LGD x discount factor ('DF')

- In a fully market consistent world, such a 'valuation' needs PD to be based on risk-neutral probabilities or equivalently DF to be a deflator

- The ideal fully market consistent way to encapsulate the risk exposures into a single **monetary** number is to use risk-neutral probabilities or the equivalent

Conclusions

- VaR vs TVaR: boils down to PD vs PD x LGD
- The mindset difference is the LGD
  - Shareholders vs. policyholders/regulators
- Treatment of illiquidity
  - LGD depends on outcomes in which the firm is **unable** to remain a hold-to-maturity investor
- Stress testing design
  - Ideally include a "test beyond destruction" element