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Extreme Events and Portfolio Construction

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Agenda

- Analysing fat-tailed behaviour
- What causes fat-tailed behaviour?
- Selection effects – an example of model risk
- Portfolio construction in the presence of fat tails

- Talk based on material in:
  – **Kemp, M.H.D. (2010). *Extreme Events: Robust Portfolio Construction in the Presence of Fat Tails*. John Wiley & Sons (forthcoming)**

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Extreme events: Robust portfolio construction in the presence of fat tails

- Chapters:
  1. Introduction
  2. Fat tails – in single (i.e. univariate) return series
  3. Fat tails – in joint (i.e. multivariate) return series
  4. Identifying factors that significantly influence markets
  5. Traditional portfolio construction techniques
  6. Robust mean-variance portfolio construction
  7. Regime switching and time-varying risk and return parameters
  8. Stress testing
  9. Really extreme events
- Plus Principles (Chapter 10) and Exercises (Appendix)
- Toolkit available through www.nematrian.com

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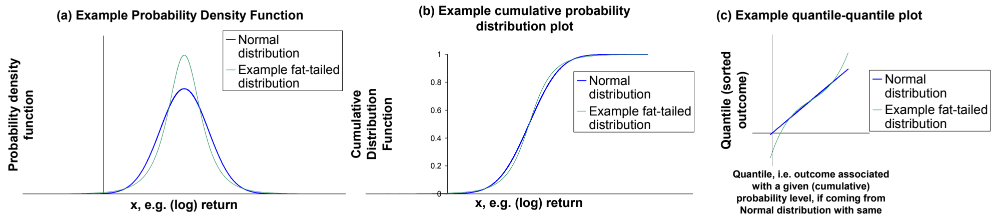
Analysing fat-tailed behaviour

- There are various ways of visualising fat tails in a *single* return distribution. Easiest to see in format (c) below
- By 'fat tail' we mean probability of extreme-sized outcomes (returns / movements / events) seems to be higher than from (log) Normal distribution



Source: Nematrian (illustrative)

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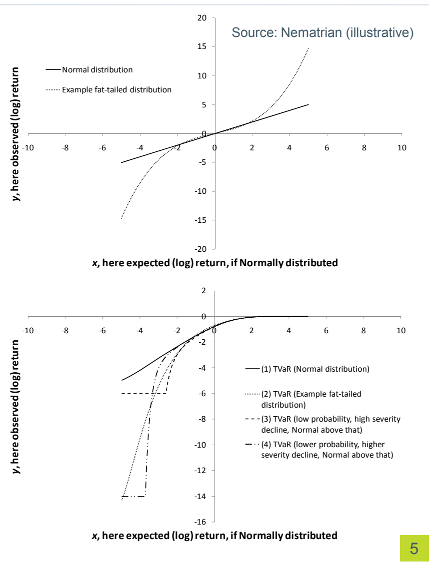
QQ-plots

- Largest divergences relate to extreme events
  - Usually what we want
- However, could wrongly emphasise extreme events
  - Under-emphasise: VaR vs TVaR
  - Over-emphasise: fat tails can add rather than subtract value



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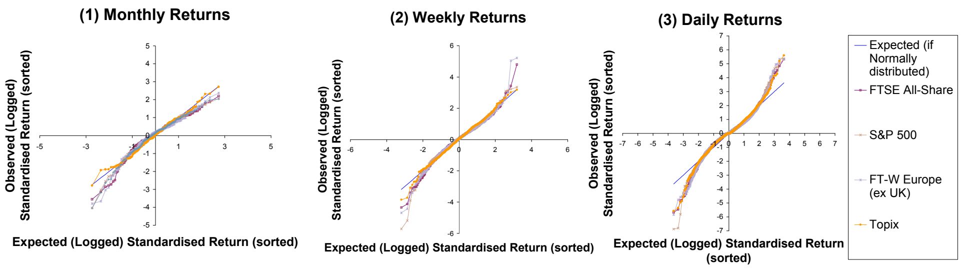
Tail behaviour dependent on time-scale (1)

- Higher frequency data
  - Typically viewed as more fat-tailed than lower frequency data
- Period analysed below: June 1994 to December 2007



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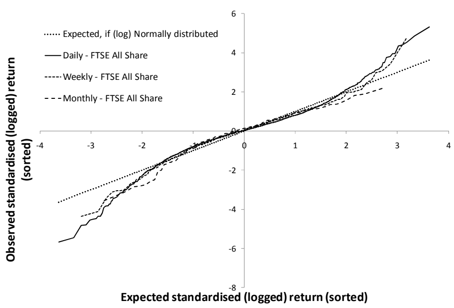
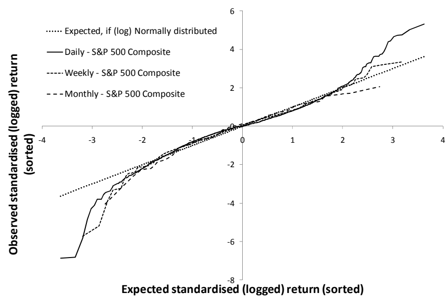
Tail behaviour dependent on time-scale (2)

- Higher frequency data
  - More data points => QQ-plot is naturally further into the tail
  - For these data sets, daily data not much more fat-tailed than weekly data
  - But note e.g. Oct 1987





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Skew(ness), kurtosis and Cornish-Fisher

- Fat tails involve deviation from Normality
  - Hence some higher *cumulants* (moments), aka *semi-invariants*, e.g. skew and (excess) kurtosis, deviate from zero (Normality)
- Cornish-Fisher (4th moment version) estimates distributional form from merely the first 4 moments, i.e.

$$mean = \mu = E(x) \qquad standard\ deviation = \sigma = E\left((x-\mu)^2\right)$$

$$skew = \gamma_1 = E\left(\left(\frac{x-\mu}{\sigma}\right)^3\right) \qquad (excess)\,kurtosis = \gamma_2 = E\left(\left(\frac{x-\mu}{\sigma}\right)^4\right) - 3$$

  - Regularly appears in risk management academic literature
- Standardised QQ-plot estimated via a cubic equation:

$$y_{CF4}(x) = x + \frac{\gamma_1\left(x^2-1\right)}{6} + \frac{3\gamma_2\left(x^3-3x\right) - 2\gamma_1^2\left(2x^3-5x\right)}{72}$$

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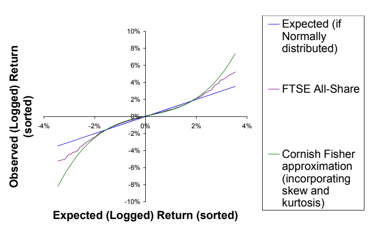
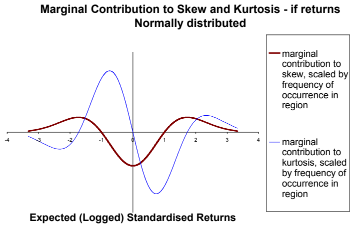
Flaws in Cornish Fisher (and hence in skew/kurtosis)

- Doesn't model index return distributions particularly well
  - Particularly parts risk managers might be most interested in, i.e. downside tails
- Computation gives less weight to tail observations (most observations are in middle of the distribution)
- Lacks a desirable stability criterion
  - Applying CF twice can lead to a more extreme distribution
- Fit QQ-plot directly, e.g. with cubic (or other weightings)?





Source: Nematrian, Threadneedle, FTSE, Thomson Datastream

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Joint fat-tailed behaviour

- Usually split between
  a. Marginals
  b. Copula
- Facilitates Monte Carlo simulation
- But some disadvantages
  – Fat-tailed characteristics difficult to see (copulas akin to joint pdf / cdf)
  – Many problems depend on (a) and (b) in tandem
- Kemp (2010) proposes a multi-dimensional variant of QQ-plots to circumvent these difficulties

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What causes fat-tailed behaviour?

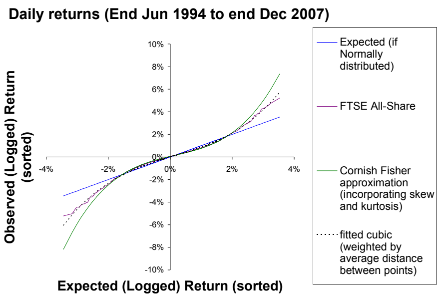
- Time varying volatility (and other distributional characteristics)
- Regime switching
- Crowded trades and leverage

Time-varying volatility

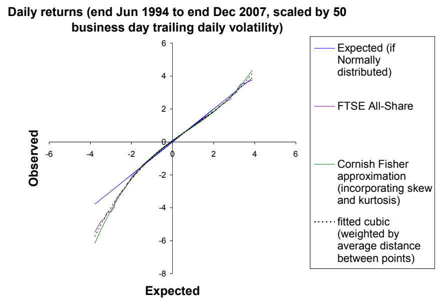
- Very widely observed phenomenon
  - Fits our intuition – sometimes markets more turbulent than at other times
- Distributional mixtures of Normal distributions
  - E.g. draw X_1 with probability p from N_1, draw X_2 with probability (1-p) from N_2
  - Quite different behaviour to linear combination mixtures, i.e. $a.X_1 + b.X_2$
- If N_1 and N_2 have same mean but different standard deviations then distributional mixture fat-tailed (if $p \neq 0$ or 1) but not linear combination mixture
- Time-varying volatility creates an analogous effect
  - Because drawing from different distributions at different times

Explains some market index fat tails, particularly on upside

Raw Data



With Short-term Volatility Adjustment



Average extent to which tail exceeds expected level (average of 6 most extreme outcomes)				
	Downside (%)		Upside (%)	
	Unadj	Adj for vol	Unadj	Adj for vol
FTSE All-Share (in GBP)	54	41	42	3
S&P 500 (in USD)	68	70	50	7
FTSE Eur ex UK (in EUR)	48	53	54	-3
Topix (in JPY)	54	72	42	39

Source: Nematrian, Threadneedle, FTSE, Thomson Datastream

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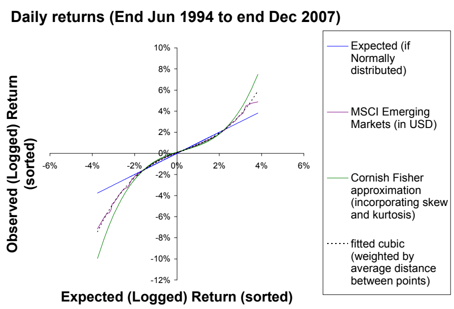
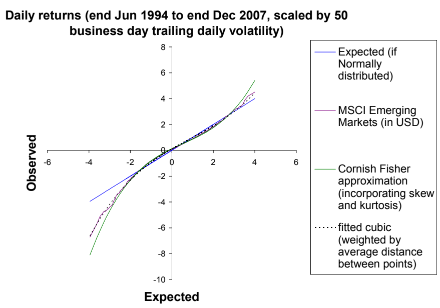
Not just a developed market phenomenon

Raw Data



With Short-term Volatility Adjustment



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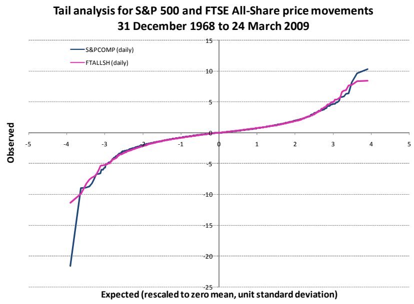
A longer term phenomenon too

Raw Data



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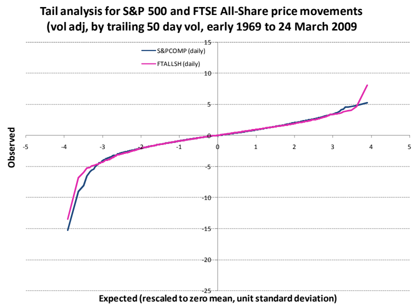

Source: Nematrian, Threadneedle, S&P, FTSE, Thomson Datastream

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Time-varying volatility

- Also known as heteroscedasticity
- Closely allied with GARCH modelling
  - E.g. $s(t) = a.s(t\text{-}1) + c$, where s = volatility (if using $AR(1)$ model)
  - The C in GARCH is because we are talking about the volatility conditional on the current time and/or on volatility at earlier times
- Why not incorporate time-varying behaviour in distributional parameters including means and correlations (covariances)?
- More commonly then called regime switching

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Regime switching

- Idea: two or more 'regimes' (each e.g. characterised by a complete $N(\underline{\mu},\mathbf{V})$ distribution, say R_1 and R_2
- World is in one of these states at time *t*
- Switches from R_i to R_j with probability $p_{i,j}$ at time *t*
  - Usually adopt a 'simple' Markov chain formulation, in which $p_{i,j}$ does not depend on what regimes the world was in before the last time period
- Can be generalised to continuously varying distributions, and continuous time
  - If latter then typically solved using stochastic calculus
  - Numerical solution typically reintroduces time grid

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Regime switching (continued)

- Adds complexity and therefore sophistication
  - And risk of over-fitting, i.e. lack of parsimony!
- Regimes might be Normal but have different means e.g. 'normal' and 'bear' regimes of Ang and Bekaert (2004)
  - Can introduce fat tails and conditional tail correlation effects
- In general, risk-return trade-off dynamics are altered
  - Optimal (i.e. efficient) portfolios then regime dependent
  - Also time dependent (and hence more sensitive to transaction costs)
  - Also utility dependent, both re. fat tails and re. inter-temporal utility

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Crowded trades

- Some fat tails still seem to come "out of the blue"
  - E.g. Quant funds in August 2007
  - Too many investors in the same crowded trades? Behavioural finance implies potentially unstable
  - For less liquid investments , impact may be via an apparent shift in price basis
- Portfolio and system-wide equivalents via leverage?
  - Leverage introduces/magnifies *liquidity* risk, *forced unwind* risk and *variable borrow cost* risk

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Selection effects, see e.g. Kemp (2010a, 2010b)

- 'Selection' effects are a common problem in finance
  - E.g. Individuals buying annuities typically have longer life expectancies than individuals who don't
- Can also apply to portfolios being analysed by risk models
  - Many risk models assume behaviour that is (approximately) Gaussian, i.e. multivariate (log) Normal, akin to lots of different sources of random noise
  - Can decompose multiple series return data into *principal components*, the most important of which contribute the most to the aggregate variability exhibited by securities in the relevant universe
- But what if portfolios are structured to seek 'meaning' (e.g. if they are actively managed!) and 'meaning' is (partly) associated with non-Normality?
  - Both *meaning* and *magnitude* are important

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Selection effects are potentially very important

	PCA, only StdDev (*c* = 0)			**Blended (*1 in 200 quantile level, CF4*)**			**ICA, Only Kurtosis**	
Component	StdDev	Kurt	Criterion	StdDev	Kurt	Criterion	StdDev	Kurt
1	10.6%	3.1	10.6%	8.3%	14.9	56.6%	4.5%	24.2
2	6.5%	2.1	6.5%	4.9%	24.9	52.7%	4.2%	23.5
3	5.6%	1.7	5.6%	5.0%	22.1	48.0%	4.5%	18.1
4	4.8%	1.4	4.8%	4.5%	14.7	30.1%	6.9%	16.2
5	4.2%	0.4	4.2%	4.3%	15.0	29.7%	4.2%	15.0
6	3.7%	1.1	3.7%	4.8%	9.2	22.1%	4.2%	13.7
Av (top 6)	5.9%	1.6	**5.9%**	5.3%	16.8	**39.9%**	4.7%	18.5
Av (all 23)	3.2%	1.2	3.2%	3.6%	8.2	17.5%	3.7%	9.1

- (a) Principal components analysis – focuses on standard deviation, (b) independent components analysis – focuses on, say, kurtosis, or (c) blend
- Sizes of '1 in 200' events potentially underestimated several-fold by PCA (and hence traditional risk systems), if factors expressed are selected for fat-tailed characteristics

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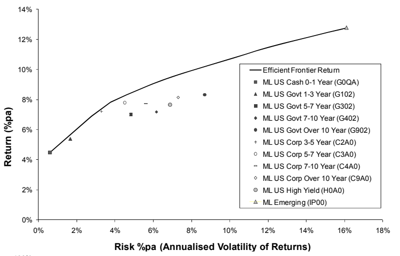
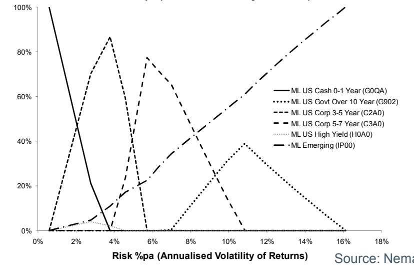
Portfolio construction

- Traditional (quantitative) approach involves portfolio optimisation
  - Typically mean-variance optimisation
  - Identify expected return ('alpha') from each position
  - Maximise expected return for a given level of risk (subject to constraints, e.g. weights sum to unity)
  - Maximise $\mathbf{a.r} - \lambda.\mathbf{a}^T\mathbf{Va}$
- Time-varying parameters add realism and complexity
  - alpha + beta 'separation'





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Portfolio construction – sensitivities

- Output results notoriously sensitive to input assumptions
- Possible responses:
  - Treat quant models with scepticism (the fundamental manager's approach?)
  - Use 'robust' approaches, Bayesian priors/anchors, e.g.:
    - Black-Litterman
    - 'Shrinkage'
    - Position limit 'priors' (e.g. 1/N, long-only, etc.)
  - Focus on reverse optimisation

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Portfolio construction – impact of fat tails (1)

- If all return opportunities (and combinations of them) 'equally' fat-tailed, then end results the same, if risk budget adjusted appropriately
- If different combinations exhibit differential fat-tailed behaviour then in principle adjust portfolio construction to compensate:
  - If we can reliably estimate these differentials
  - And if investors do not have solely quadratic utility functions

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Solution A - simplest

- Most important (predictable) single contributor to fat tails seems to be time-varying volatility. So:
  - Calculate covariance matrix between return series after stripping out effect of time-varying volatility
  - Optimise as you think fit (standard, “robust”, Bayesian, BL, ...), using adjusted covariance matrix
  - Adjust risk aversion/risk budget appropriately
  - Then unravel time-varying volatility adjustment
  - Or derive implied alphas using same adjusted covariance matrix
- Implicitly assumes all adjusted return series ‘equally’ fat-tailed

Solution B – more sophisticated

- Model with a mixture of multivariate Normal distributions
- Time-stationary? Maybe not realistic?
- Time-varying?
  - (Discrete) regime switching, and/or
  - (Continuous) parameterisation (and continuous time?)
- However:
  - Even a mixture of just two multivariate Normal distributions involves estimation of twice as many parameters
  - Making parameter estimation correspondingly less reliable
  - Results very sensitive to input assumptions
  - Time varying => dynamic => sensitivity to transaction costs

Solution C – lower partial moments

- Any return = threshold + upside + downside
- Non-quadratic utility will typically give greater weight to downside and will in general also depend on higher moments
- Single series, define as: $lpm(K,m)$=E[min($(r-K)^m$,0)]?
- Multiple series, define as: $lpm_{i,j}(K,m,n)$= E[min($(r_i-K)^m(r_j-K)^n$,0)]?
  - Or max
  - E.g. co-skewness, co-kurtosis
  - Or symmetric alternatives
- Substantially increased numbers of parameters, and few observations in tail
  - Specify candidate distributional form and fit this?

Summary

- Fat-tailed behaviour
  - Very common in practice
  - Several intrinsic reasons for its existence, including time-varying world
  - QQ plots focus more on extremes than pdf /cdf
- Active management may 'select' fat-tails
  - Potentially major implications for risk modelling
- Portfolio construction can be refined to cater better for extreme events
  - But refinements potentially complex, especially in a time-varying world

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